Catch a Star 2008

How speedy are planets?

Measurements of radial velocities in the spectra of bright planets

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1. Abstract: Who we are and what the aim of our entry is

As passionate members of the astronomy group of our local high school and fascinated by spectroscopy and its fundamental importance for today's knowledge about stars and galaxies, we were soon determined to dive further into the world of spectral lines and infinitesimal electromagnetic radiation. In order to combine this new topic with our knowledge about the solar system (we took part in CAS 07 last year with our topics of the Sun and Saturn), we have been analyzing the spectrum of sunlight reflected by several planets, carefully supervised by our teacher.

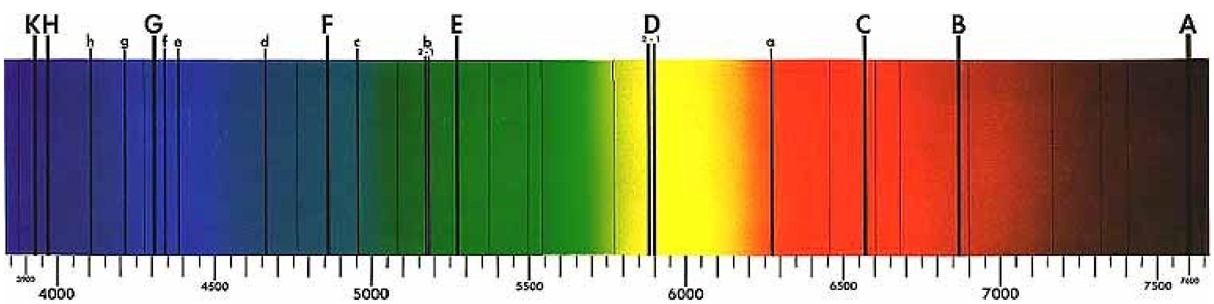
Our project is based on the central idea that there will be Doppler-shifts of wavelengths in these spectra due to the relative movements between the planets and our Earth. We assumed, however, that only 'normal' absorption lines, the so-called Fraunhofer lines, will be shifted, but not the so-called telluric lines that result from the influence of the earthern atmosphere. The latter therefore can serve as reference lines for the precise determination of the Doppler shifts. Thus, you can determine the relative velocity of the Earth and the respective planet.

The aim of this work is to present how we analyzed the spectra of the Sun itself and the planets with the help of a high-resolution grating spectrometer and then looked for appropriate telluric lines and Fraunhofer lines, not omitting the physical background of the observed processes.

We chose Venus and Jupiter as the planets we wanted to observe because they offered the best conditions in the period of our project. The observations took place at our school-observatory some 40 km south of Hamburg, Germany.

2. Fraunhofer lines

The Fraunhofer lines[1] were named after their discoverer and appear as a result of absorption. The absorption is a result of the gases contained in the photosphere absorbing a specific wavelength of the sun light. This is the reason why no complete continuum spectrum of the Sun is visible. There, where the spectrum shows gaps, dark lines are visible. These absorption lines are called Fraunhofer lines. It needs to be mentioned that the assumption that all these absorption lines have the same shade of darkness is not true. With respect to the width and depth of a line, this "darkness" is rather a specific characteristicum. The reason is that, ultimately, the intensity of the Fraunhofer lines as a result of the dependency on the chemical composition of the absorbing gas mixture is reduced in such a way that the naked eye is not able to exactly perceive the fine differences in brightness. This phenomenon – that the intensity is different for each gas mixture – allows the scientists to draw conclusions regarding the chemical composition of the photosphere. Therefore, today, we are able to say which gases are and which gases are not part of the photosphere. Without this discovery, our knowledge of the Sun would be much less sophisticated than it is now.



A graphic display of important Fraunhofer lines marked in

3. Telluric lines

However, not all the absorption lines we see in the spectrum of the Sun come into being in the Sun's photosphere. In fact, while the sunlight passes the Earth's atmosphere, some photons are absorbed by 'earthern' molecules as well. Absorption lines that stem from the influence of our planet's atmosphere are called telluric lines. For instance, three of the most important Fraunhofer lines (A, a and B in the red side of the spectrum) derive from 'earthern' gaseous oxygen and water vapour respectively. [2]

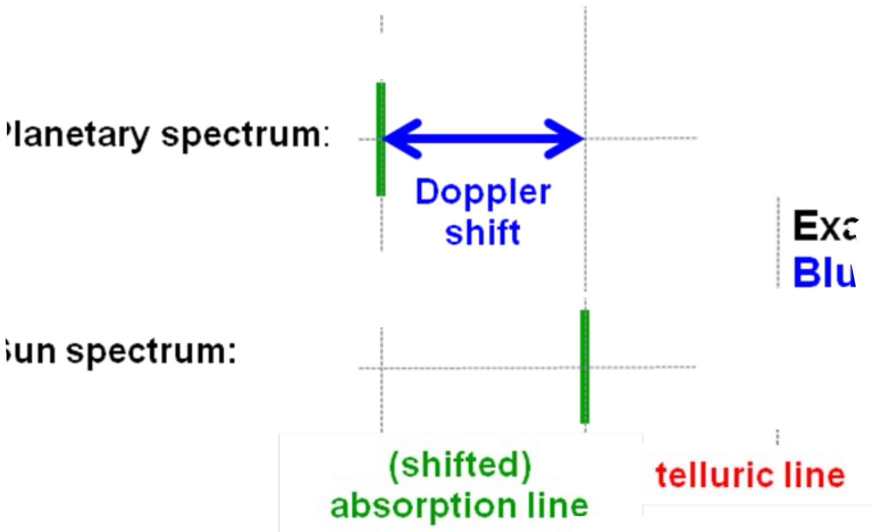
Note that short before sunset, when the way of the sunlight through the Earth's atmosphere is much longer, more photons are absorbed. That is why the telluric lines at this time are stronger than at midday.

Let us emphasize that in contrast to 'normal' absorption lines the wavelength of telluric lines are constant, they are not shifted due to the Doppler effect. In other words:

[1] cf. Kaler: Sterne und ihre Spektren (Stars and their spectra), p. 85f.; following illustration of Fraunhofer lines: http://de.wikipedia.org/wiki/Bild:Fraunhofer_lines.jpg

[2] cf Kaler: Sterne und ihre Spektren. S. 85f.

However fast the Earth moves relatively with respect to planets or stars, its 'relative velocity' towards its own atmosphere is always absolutely zero.
Therefore telluric lines can serve as reference lines[3] for our Doppler experiments:
If you compare the distance between the original wavelength of a spectral line and the wavelength you measured regarding such a reference line, you can calculate the Doppler wavelength shift and thus the radial velocity (cf the following chapters):



4. Doppler effect

We know this phenomenon from everyday life, when an ambulance with its siren going approaches you: You will hear a higher frequency with an immediate down-tuning when the car passes you and recedes. In general you can illustrate the Doppler effect with the help of the following thought experiment:
Assuming a stationary source emits electromagnetic or acoustic waves whose frequency is measured by a likewise stationary observer. The wavelength λ can then be expressed as:

$$\lambda = c \times T \qquad \text{(T: wave period)} \qquad (1)$$

Now let the observer approach the source of the waves or vice versa, then he will certainly count more waves per time unit (that is, a higher frequency, i.e. a shorter wavelength) than before. If he moves with the velocity v, the original wavelength is reduced by $\Delta\lambda = v \times T$ (cf illustration):

$$\lambda' = \lambda - \Delta\lambda \quad \leftrightarrow \quad \lambda' = \lambda - v \times T \qquad (2)$$

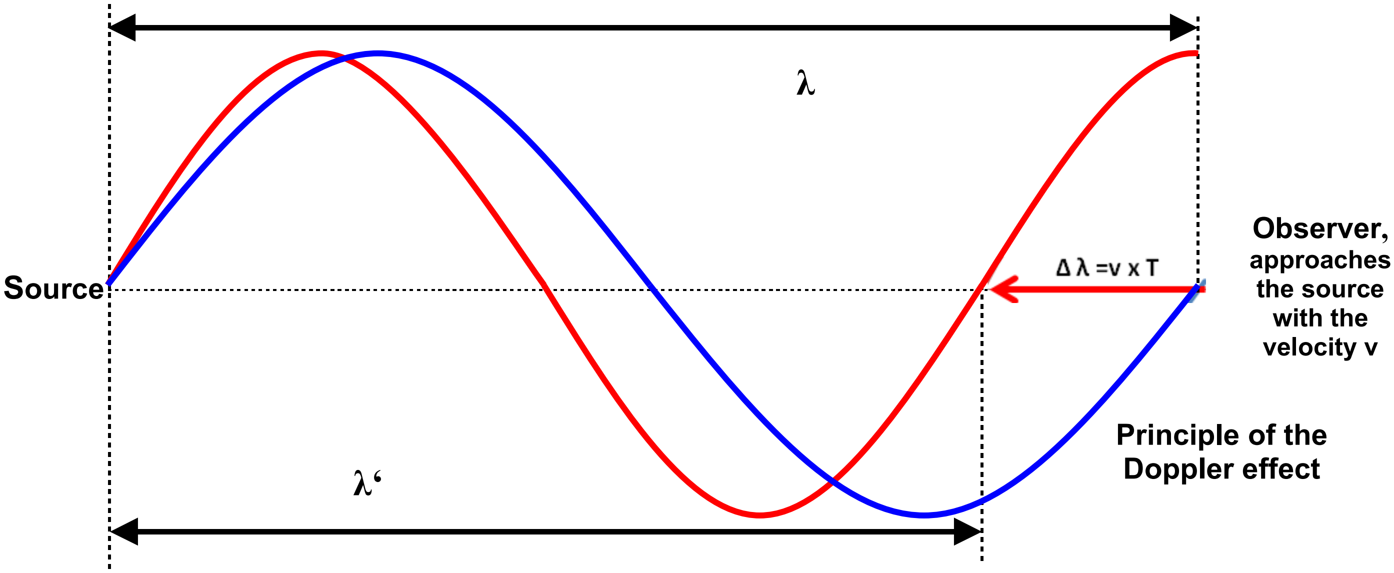
Writing v alone on one side of the term, we find:

$$v = \frac{\lambda - \lambda'}{T} \qquad (3)$$

[3] References in astro-spectroscopy are normally based on laboratoy wavelengths of emission lines from expensive gastubes. As a help and crosscheck we made use of a Neon-lamp from a light switch after defining the lines with the help of the Internet.

Regarding (1) this can be expressed as

$$v = \frac{\lambda - \lambda'}{\lambda} \times c \qquad \leftrightarrow \qquad v = \frac{\Delta\lambda}{\lambda} \times c \qquad (4)$$



Hence, an absorption line in the spectrum of Venus will be shifted to the blue side of the wavelength scale if the Earth relatively approaches Venus (**blueshift**: the radial velocity is negative), otherwise (that is, if it recedes relatively from Venus) there will be a **redshift** in the planetary spectrum (then λ' is tantamount to $\lambda + v \times T$ and the relative velocity is positive). (Do not confuse the Doppler redshift with the redshift stemming from the expansion of space!).
According to the Doppler formula the shift of a spectral line Δλ is proportional to the relative velocity v, in other words: the higher the amount of v, the higher the blue/redshift.[4]

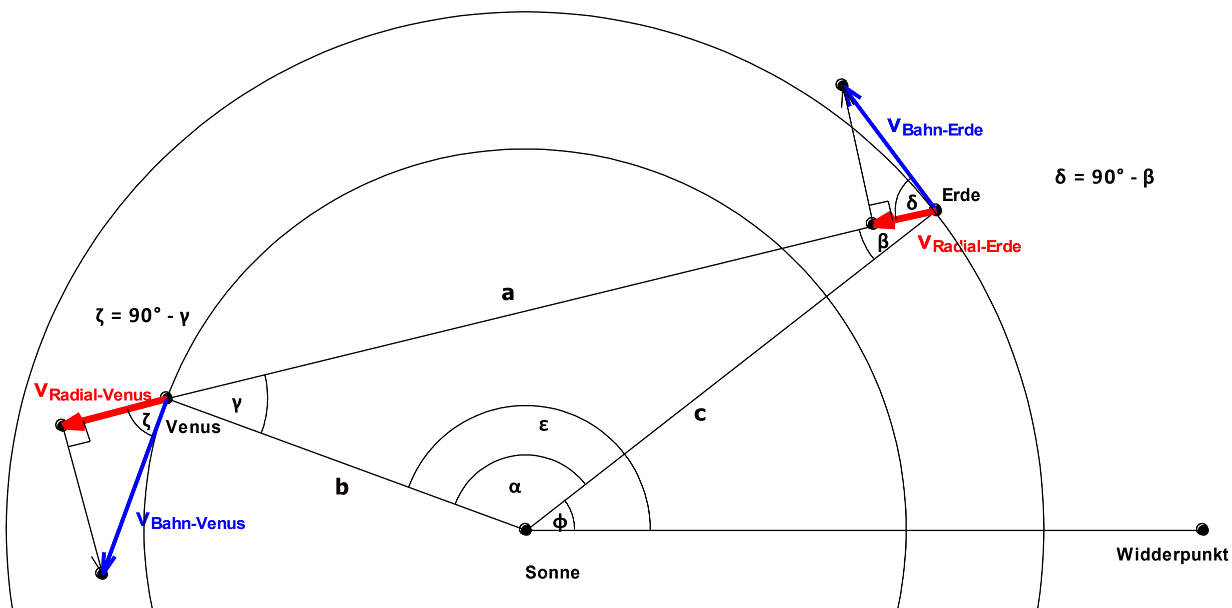
5. Radial velocity

In our case the relative velocity of the Earth with respect to other planets is the vector difference between the radial velocities of both the Earth and the planet. The radial velocity in general is the component of a star's / planet's movement in direction of the Earth (and therefore causes Doppler shifts in their spectra).

[4] For the Doppler effect see http://leifi.physik.uni-muenchen.de/web_ph12/versuche/12dopplereffekt/dopplereffekt.htm

If you know the original wavelength of a spectral line from laboratory experiments you can calculate the Doppler shift by subtracting the wavelength you measured from the 'real' one. Using the Doppler formula you then can calculate the relative, i.e. in this case the radial velocity of a star (see illustration in chapter 'telluric lines').
Radial velocities are of the utmost importance for our knowledge of the movements of stars and galaxies (e.g. their rotational and escape velocities). They are especially significant concerning binary stars that can only be distinguished by spectroscopic means.[5]

For deriving a formula to calculate the relative velocity of the Earth with respect to Venus depending on the Earth's position let us consider the following illustration:



System Sun – Earth – Venus (German sketch)

As seen above the relative velocity is the difference between the radial velocities of both the Earth and Venus (since it decreases if both have the same direction, you have to subtract, not add!): $v_{Relative} = v_{Radial\text{-}Venus} - v_{Radial\text{-}Earth}$ (5)

In order to portray this context in simplified terms we chose the following reductions:

- Due to the low eccentricities of both planets (Venus: 0,0068; Earth: 0,0167)[6] we replaced the actually elliptic orbits with ideal circular ones and regarded the orbital speed of them as constant.
- Due to the low inclination of Venus towards the ecliptic (3,4°) we only regarded degrees of longitude (angle to the connecting line of Sun and the so-called vernal point, cf illustration).

[5] cf Kaler: Sterne und ihre Spektren. S. 34, 72, 192f. and 197.
[6] For the data of the orbital elements used in the derivation see http://de.wikipedia.org/wiki/Planeten_des_Sonnensystems_%28Tabelle%29

In the triangle Sun-Venus-Earth there are the following distances:

Sun-Venus b:	0,7233 AU	(semi-major axis of Venus)
Sun-Earth c:	1 AU	(semi-major axis of the Earth)

and the following orbital speeds:

Venus:	35,02 km/s
Earth:	29,79 km/s

According to the law of cosines the distance between the Earth and Venus can be expressed as:

Venus-Earth a: $\sqrt{(b^2 + c^2 - 2 \times b \times c \times cos\alpha)}$ (6)

There are the following correlations between the angles:

$180° = \zeta + \gamma + 90° \leftrightarrow \zeta = 90° - \gamma$ and
$90° = \delta + \beta \leftrightarrow \delta = 90° - \beta$

Using trigonometric correlations we find for the radial velocities of the two planets:

$$v_{Radial\text{-}Venus} = v_{Orbital\text{-}Venus} \times \cos\zeta \leftrightarrow v_{Radial\text{-}Venus} = v_{Orbital\text{-}Venus} \times \cos(90° - \gamma)$$

$$v_{Radial\text{-}Earth} = v_{Orbital\text{-}Earth} \times \cos\delta \leftrightarrow v_{Radial\text{-}Earth} = v_{Orbital\text{-}Earth} \times \cos(90° - \beta) \quad (7)$$

With the help of the law of sines and the sum of angles in a triangle we can calculate the missing angles, e.g.
$\gamma = \sin^{-1}(\frac{c \times \sin\alpha}{a})$

Using the law of sines and (6) we can write

$$v_{Radial\text{-}Venus} = v_{Orbital\text{-}Venus} \times \cos\left[90° - \sin^{-1}\left(\frac{c \times sin\alpha}{\sqrt{b^2 + c^2 - 2 \times b \times c \times cos\alpha}}\right)\right] \text{ and}$$
$$v_{Radial\text{-}Earth} = v_{Orbital\text{-}Earth} \times \cos\left[90° - \sin^{-1}\left(\frac{b \times sin\alpha}{\sqrt{b^2 + c^2 - 2 \times b \times c \times cos\alpha}}\right)\right] \quad (8)$$

Now we can rewrite (5) as

$$v_{Relative} = v_{Orbital\text{-}Venus} \times \cos\left[90° - \sin^{-1}\left(\frac{c \times sin\alpha}{\sqrt{b^2 + c^2 - 2 \times b \times c \times cos\alpha}}\right)\right] - v_{Orbital\text{-}Earth} \times \cos\left[90° - \sin^{-1}\left(\frac{b \times sin\alpha}{\sqrt{b^2 + c^2 - 2 \times b \times c \times cos\alpha}}\right)\right] \quad (9)$$

As $\alpha = \varepsilon - \varphi$ α contains the Earth's position we have finally reached our aim!!
Note that in general $\sin^{-1}$ is not definite! However, in this case it is indeed since β and γ always are $\leq 90°$.

By using Redshift (astronomy software) we know the angles of the planets at the time of our observation (January 13th, 2008 at 7^{00}):

φ=112.42° and ε=203.13°
The relative velocity of the Earth with respect to Venus therefore amounts to

$v_{Relative}$ = **10.85** km/s (i.e. Venus relatively recedes from the Earth)

Using the Doppler formula we can theoretically assume a redshift for an absorption line with λ=628 nm of

$\Delta\lambda = \frac{v}{c} \times \lambda$=0.0227 nm.

Let us be curious what we will measure in practice!

6. What is spectroscopy?

Everyone has seen a rainbow and wondered how it has come into existence. The sunlight falls upon a raindrop and this divides the light into its fragments, thus into its colours. The result is a rainbow, a (very low-resolution) spectrum of the sun. This is spectroscopy in its natural way.
Astro-spectroscopy in the visual part of the spectrum, with which we have been concerned with, is a serial of experimental methods to gain information about the light emitting matter through decomposition of the stream of light, called photons. Thus one can analyze the light emitting photosphere of the celestial object, its temperature, the chemical composition up to the point of the relative movement of the light source (stars, nebulae, and redshifts of galaxies, GRBs etc.). Furthermore, atoms and molecules can be identified through their characteristical spectra in comparison to earthern lab-data.

6.1. Critical items of spectroscopy

To gain a spectrum, which is sharp enough for astronomical analysis, we use a diffraction grating with a very small distance between the grooves.[7]

To explain how a spectrum comes into existence, we use the double-slit analogy:
In this experiment we understand light with wave-like properties. According to the Huygens-Fresnel-principle, there occur two wavelets behind the double-slit, which interfere with each other. The distance between the slits and the point M is the same and therefore it follows that the retardation is zero and we can see a white strip on the wall.
Departing along the screen from the point M, the wavelets have to cover different distances to a certain point on the screen and the result is a different retardation δ. When the retardation is not exactly k*λ, then the interference is destructive and we see a black spot on the screen: a minimum. Departing further from the point M, one sees again a bright spot: a maximum. Maxima and minima take turns, but if they are too far away from the point M, they are out of focus for astronomical purposes. Maxima and

[7] Cf Kaler: Sterne und ihre Spektren. S. 73-81.

minima are called orders and the point M is the maximum of the first order. From this point on one counts the maxima and minima.
A maximum comes into existence where the retardation is again k*λ and therefore the interference is constructive. The waves collide, interfere with each other and combine. This makes a spectrum visible, which shows the several elements of the expanded light.
When using two slits, the spectrum is of very low resolution. For improvement a grating with thousands of fine, groove-like depressions is used. This grating diffracts reflected light-waves/photons according to their intrinsic energies measured in wavelenghts.

6.2. Basics of an absorption spectrum

For explanation we use the hydrogen atom, because it has only one electron and therefore it is the most uncomplicated element. The electron 'orbits' around the nucleus. The electron always tries to be nearest to the nucleus, so in order to raise an electron to higher "shells" or larger orbits certain quanta of energy are needed.
Further understanding is granted when we see light as packages of photons measured in different wavelengths. If a photon with a certain wavelength collides with an electron and if the photon has exactly the energy-quantum which is necessary to raise an electron from a state m to a state n, the photon emits its complete energy (the photon can only emit its complete energy or none at all). If the photon does so, it is able to raise the electron to a higher state resulting in a lack of colour in the continuum, because the photon has emitted its complete energy and therefore there is a black (absorption) line at that spot in the spectrum.

If you want a calculation of the energy which is necessary in the hydrogen atom, we find

*E=hf [E=energy, h=4.135*10^-15eV (Planck constant), f=frequency]*

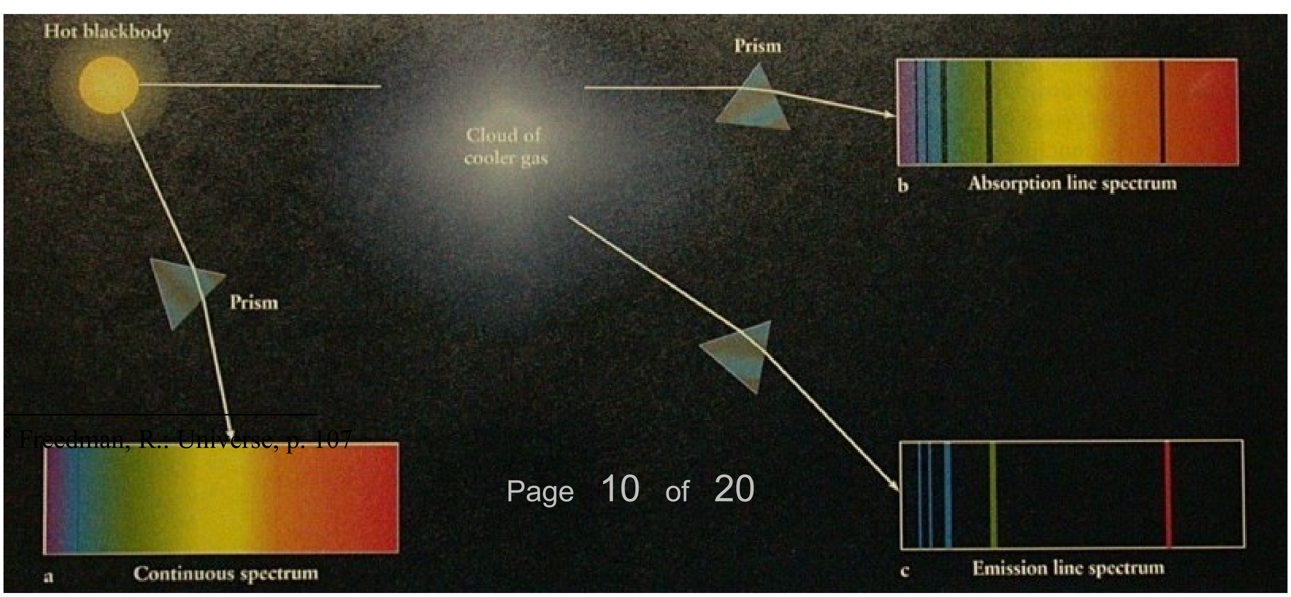
To find out the frequency of the photon, we consider:

*f=fR (1/m^2-1/n^2) [here: m=2 und n=3, fR=3.29*10^15 Hz (Rydbergfrequency)]*

*f=4.57*10^14 Hz, c=λ*f ⇒ λ=c/f=656nm, E=1.89eV*

To raise an electron of the hydrogen atom from the second to the third "orbit", one needs a photon with a wavelength of 656nm. The photon emits all its energy, so to speak 1.89eV, and the electron needs exactly this energy to get into the third "orbit".

The result of the process is shown in the following graph from an astronomy textbook[8]:



[8] Freedman, R.: Universe, p. 107

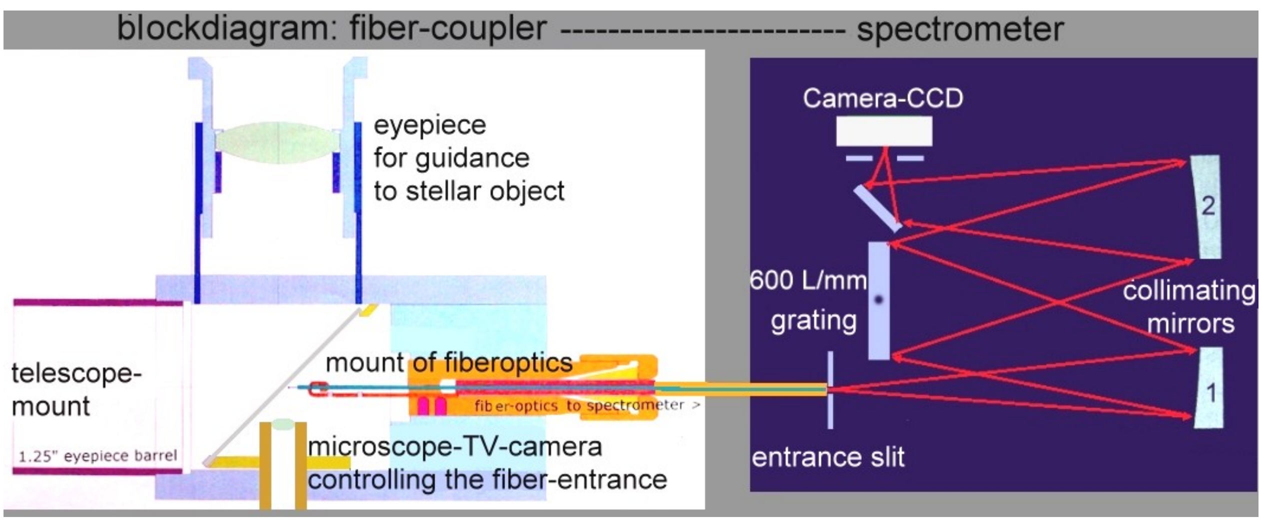
7. Observations and measurements
7.1. The technology of our spectroscopy

The spectroscopic equipment of our observatory is based on a Jobin Yvon spectrometer linked to the telescope by means of fiber-optics, which is rather costly, complicated and therefore supported by the help of Prof. Dr. em R. Ulrich from TU Hamburg-Harburg.
The spectra are recorded with a consumer DSLR-camera and a semi-professional b/w CCD-camera with a defined spectral response and effective low-light abilities (signal-noise-ratio). We list the technical data here:
Telescope: Maksutov reflector; aperture: 250mm; focal length 3250mm
(focal reducer ~0.5x); motor-driven mount (Astro-Physics 800)

Spectrometer: HORIBA Jobin Yvon (comparable to Triax 550);
focal length f 640 mm; f/5.6; resolution ~ 0,02 nm; (λ/Δλ ~30000);
diffraction grating (blazed to the second order), 600 lines/mm;
linked via fiber-optics to the telescope by a special device (s. below)

Cameras: Nikon D50/pixel-pitch: 6.8µm; Meade DSI II Pro/pixel-pitch 8.3µm
Software: IRIS; Photoshop; Corel Draw

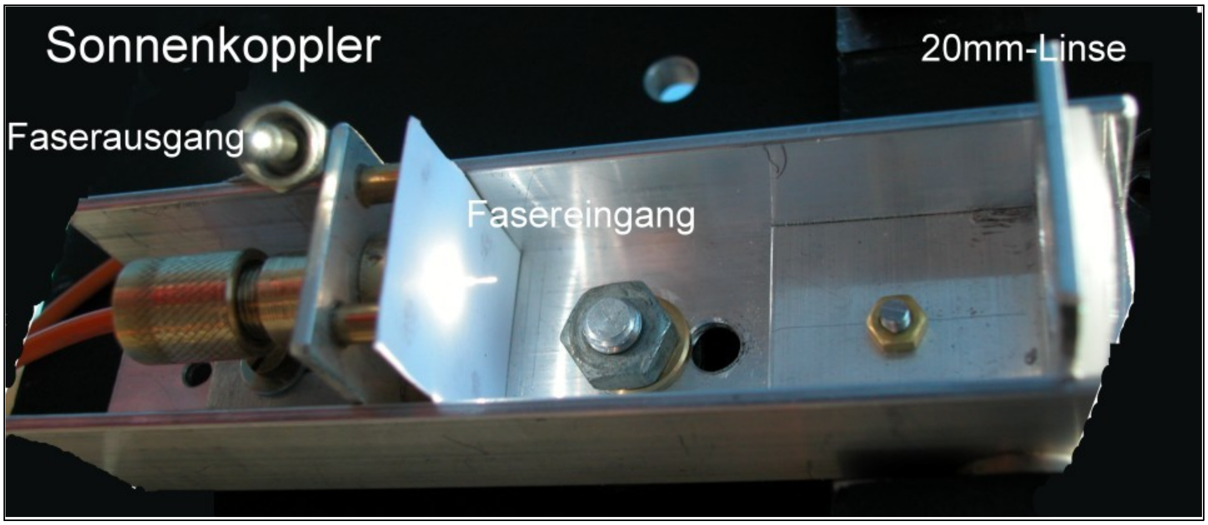
Special care has become necessary for the problem of placing the Airy-disc of the star or planet (about 10 µm in case of a star) in the focal plane of the telescope into the entrance of the fiber (100µm). Due to a relatively high wobbling in our mount drive (it amounts to ~ +/- 15 arc seconds/8min) one has to constantly supervise this process. We solved the problem with a TV-microscope-camera as shown in the following diagram (not to scale and separated by a 10m fiber):



At the diagonal sheet of glass in the fiber coupler the part of the light that is scattered while passing the sheet is used for controlling the tracking of the stellar object.

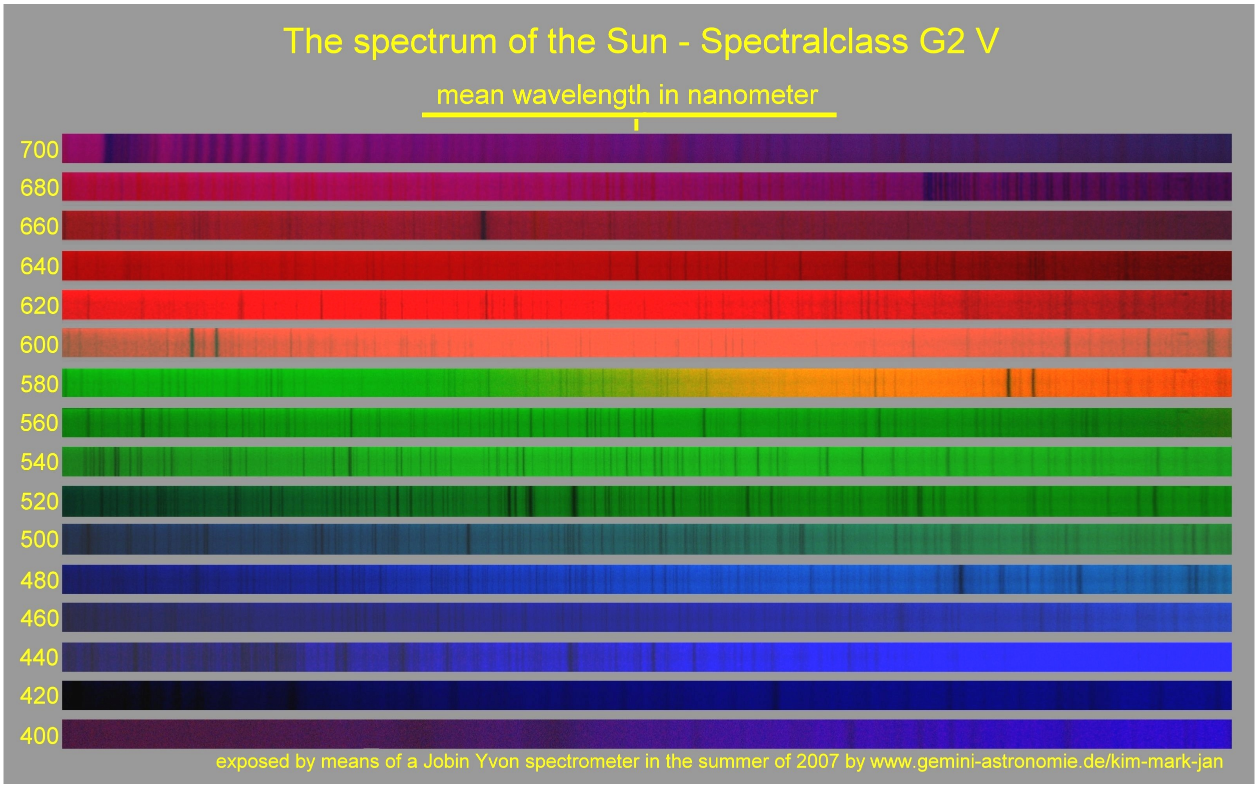
The light we receive from a planet then is directed to the entrance slit (~12 - 60 µm) of the spectrometer. The spectrum we gain on the camera-sensor (dimensions: Nikon D50:

23,6 x15,6 mm; Meade DSI: ~8x6mm) is expected to be resolved to at least ~ 0.02nm/pixel.
In the summer of 2007 we started the project by “appraoching” the Sun for a high-resolution spectrum of our “home-star”. Because of the fact that telescopes normally cannot handle the immense flux of light/energy radiated by the Sun, we needed another special device to guide the sun light into the spectrometer: With the help of a simple 20 mm plano-convex lens with a focal length of 60 mm the sun is projected directly into the fiber optics. This ‘Sun coupler’ is fixed to the motor-driven telescope mount and therefore will have the Sun in focus during the measurements:



The end of the fiber was placed onto the device for the photo showing the glare of sunlight.

Thus, we photographed the whole visual spectrum from 400 to 700 nm in steps of 10 and 20 nm respectively. Thus we got many single pictures with different times of exposure, always trying to achieve high visual sharpness and contrast of the absorption lines.
With the help of Photoshop-software we then arranged the single segments of the spectrum. As a result, similar to the publications by solar observatories in the Internet; we received the whole visible spectrum of the sun (see below):

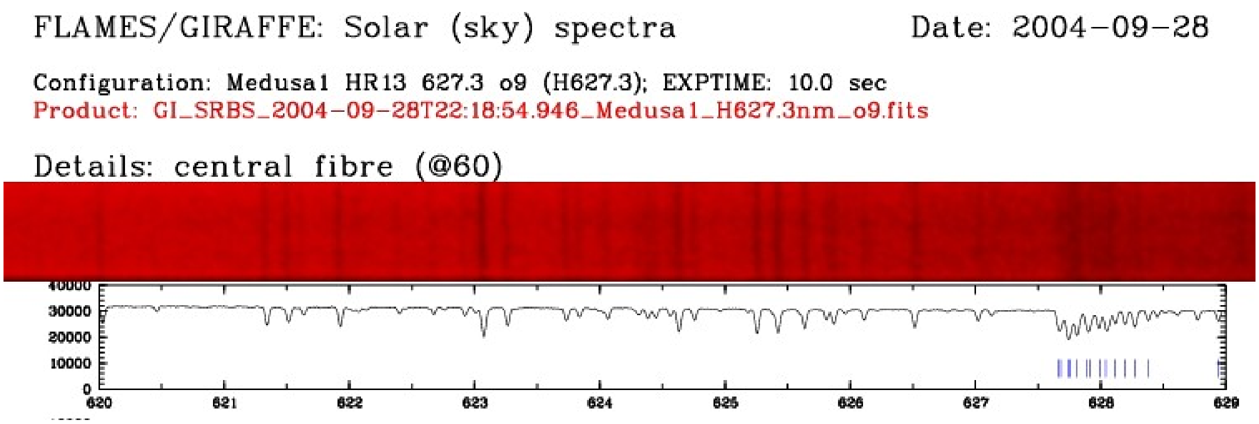


The resolution is about 0.02 nm/pixel (with an entrance slit of ~ 20 µm).

7.2. Venus' "Redshift"

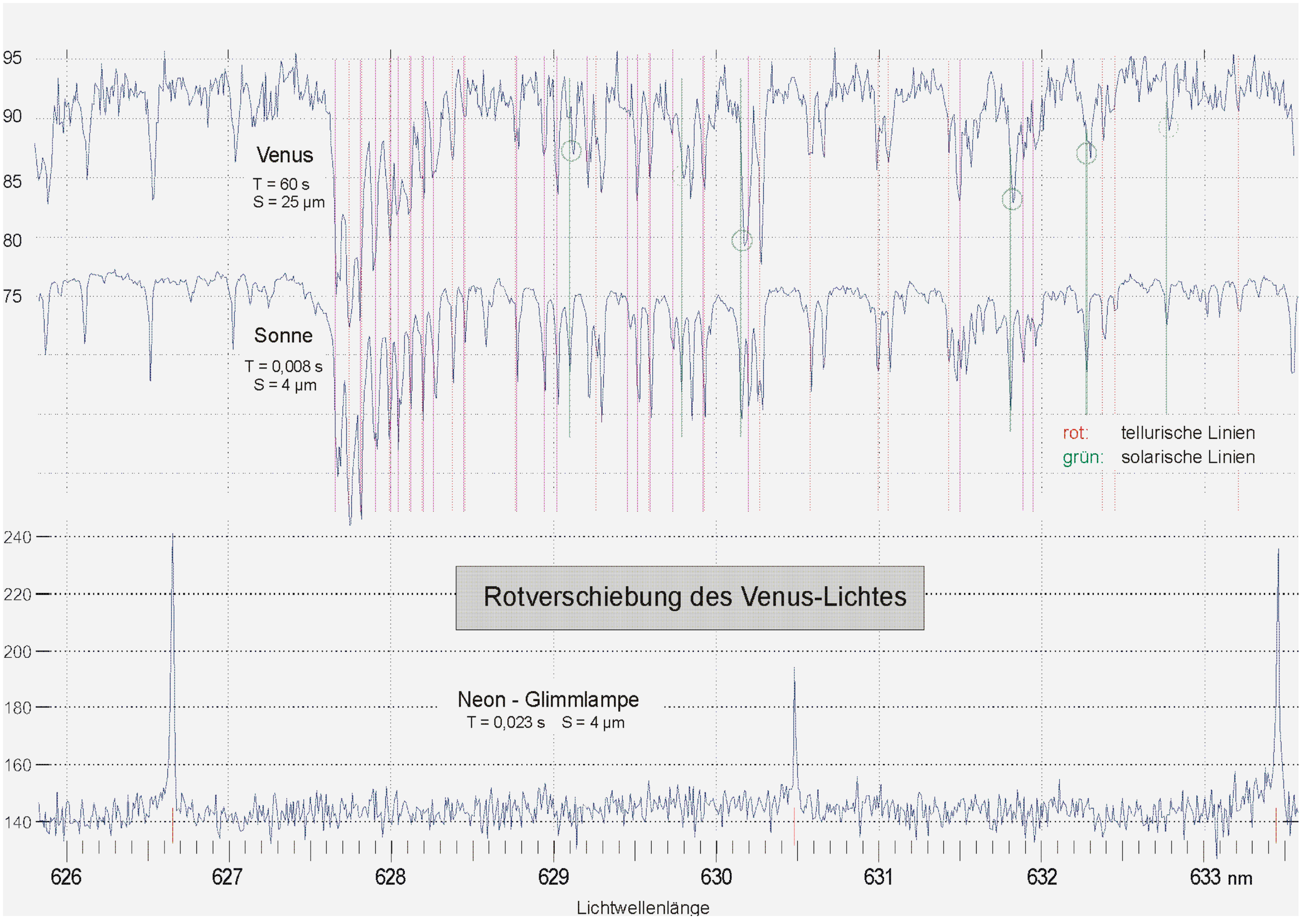
The following step was the identification of the telluric lines in the solar spectrum in order to compare them with the proper part in the Venus-spectrum. Data about the lines are perfectly provided by the ESO-Giraffe website and Hanuschik[9]. We plotted the spectral range around 625 nm with the help of IRIS software[10] and compared it to the Giraffe data, which show the well-marked telluric lines. We want to illustrate our findings here in a photoshop arrangement showing the Giraffe plot and our respective spectrum:

[9] Hanuschik – ESO 2006 (unpublished), E-Mail-Address: rhanusch@eso.org
[10] We are thankful to Mr Christian Buil, who offers not only the excellent software (for semi-professional amateurs) on his website but a tutorial as well. We learnt a lot about how to handle our spectra, which means that we would like to refer to the tutorial instead of describing the single steps in the report.
Iris. Version 5.51 2007. (Internet: www.astrosurf.com/buil)

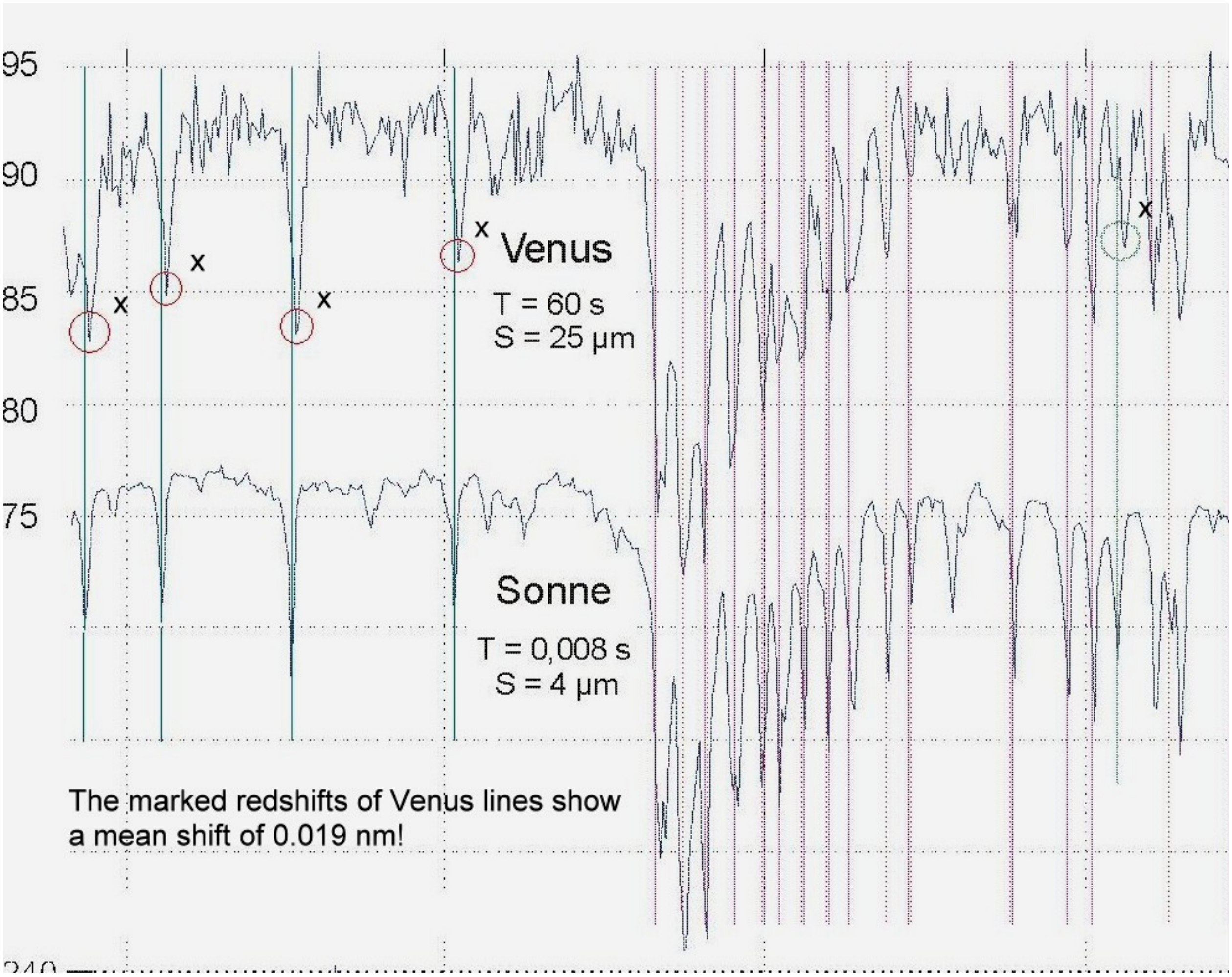


A slightly different example is here to underline IRIS' powerful pixel resolution, so that each pixel of the spectrum-plot is numbered and can be measured regarding any expected shift. We choose the Magnesium triplett around 518 nm:

During some clear nights last autumn we tested the intensity of a stellar object presented on the TV-monitor by our tracking device to find out that the display of planets was limited due to one main reason:
The sensitivity of a our TV–camera was too weak in case of planets because their magnification, together with the magnified entrance of the fiber resulted in the fact that only bright planets like Venus, Mars and Jupiter could have been trackable. Since Mars was close to its opposition this winter– when the radial velocity is close to zero – we were restricted to Venus, which we happily got into focus in the morning hours of a chilly day in January (see calculations end of ch. 5). After hours of computer work with IRIS and COREL DRAW on the plotting of the Venus shots "merged" with the respective solar lines and emission lines from the Neon light source (see ref. 3) we could finally state that we measured a respectable redshift:



Below, you can see our findings in a detail of this diagram:

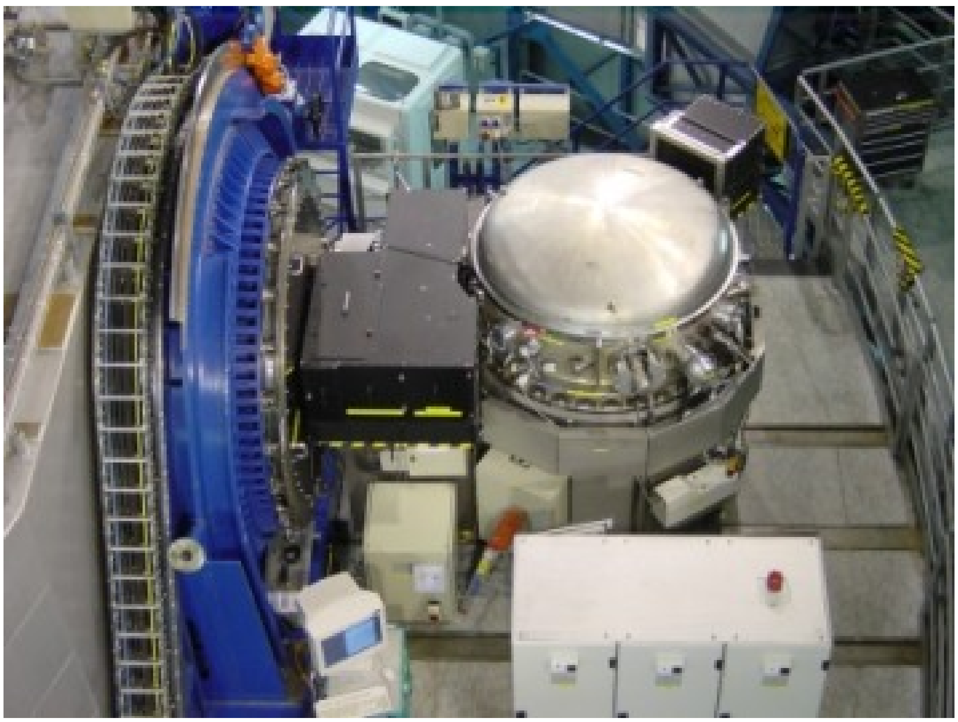


We have recorded the spectra on Jan.13th 2008 from 8^{30} to 9^{40} under the same conditions of temperature and moisture one exposure after the other together with the Neon reference line. The entrance slit was set to 25 µm for Venus and 4 µm for the Sun and the Neon emission source. As you can see, there is an average redshift of 0.019nm of the absorption lines. Comparing this with the theoretical redshift of 0.0227nm (see chapter 5), there is a difference between the theoretically calculated and practically measured redshift of only 16% (!).

8. ESO-telescope-observations related to our field of interest

Since astronomers already have an extensive knowledge of the planets' orbits and their physical parameters, the ESO-telescopes surely concentrate on other topics. From the wide range of interesting fields that ESO-telescopes are designed to explore, we would like to mention some aspects closely related to our work:

- As we are limited to the visual part of the electromagnetic spectrum, we were surprised to find H.U. Käufl's (et al) calculations of the ESO infrared spectrograph CRIRES, designed to break the limits of resolution in the infrared-spectroscopy: The center of an absorption line in the infrared (λ~1-5μm) will be resolved to a precision of ~20 m/s (!) after the photon of a, say, 10 mag celestial body, (whose Doppler shift is caused by a “warm” exosolar planet) has been reflected 27 times (!) inside the spectrograph. The giant instrument, cooled down to ~65 K, is placed in the Nasmyth-focus of one of the 4 VLTs and offers “short” integrating exposure times of about 1.5 h.

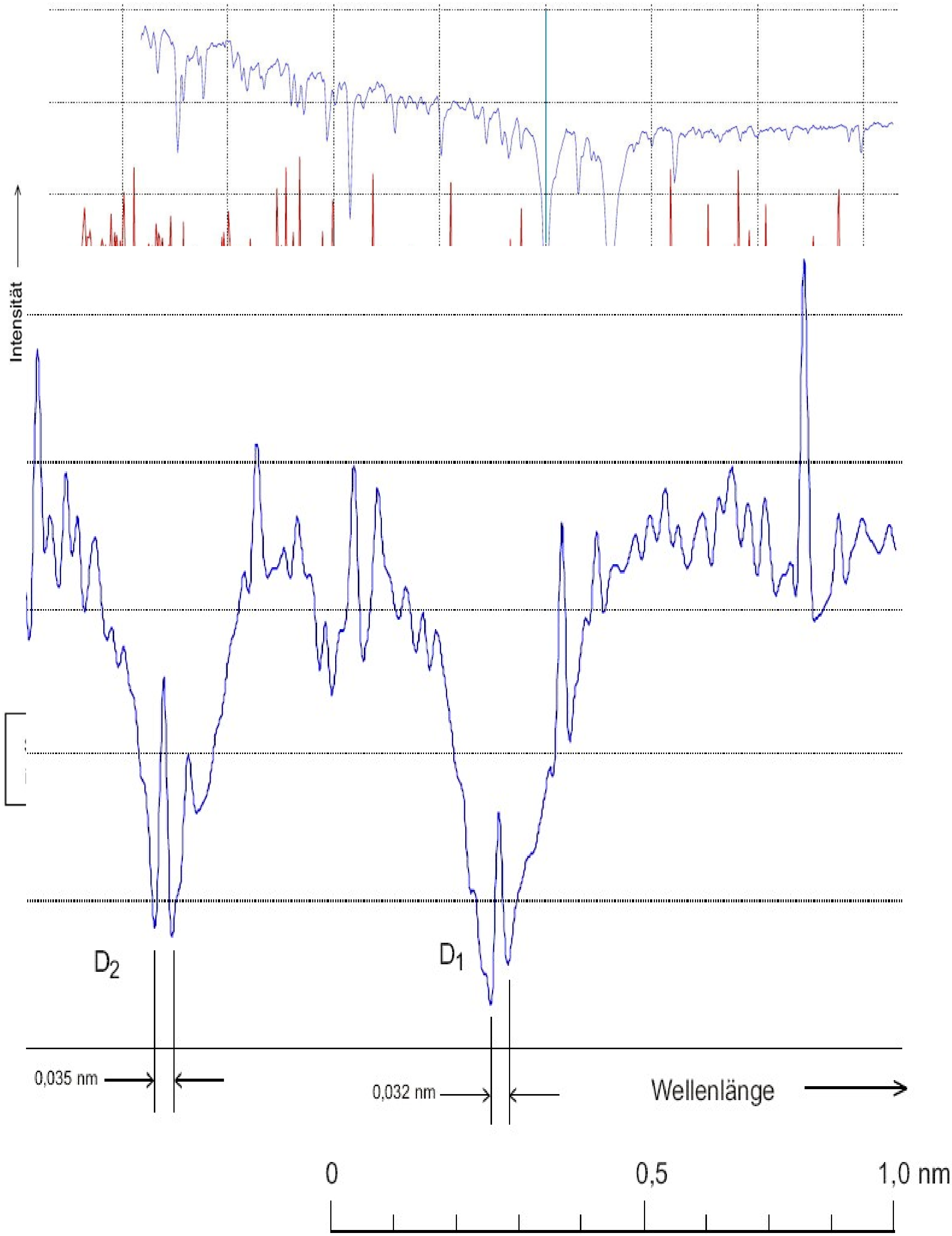


From ESO CRIRES site; note the size of the computer monitor!

- Using one of the VLTs and far better spectrographs than ours like CRIRES one could find out more about the atmospheres of our solar system planets' moons. In the case of Io (Jupiter) and Enceladus (Saturn) which have got atmospheres due to giant volcanoes it should thus be possible to find out what their volcanoes exactly spew out.
- Another field of activity concerning exosolar planets might be to analyze the spectra of stars during the transition of the planet so that the light has to pass the planet's atmosphere. Comparing these spectra with the stars' 'normal' ones, one could learn more about the composition of these atmospheres, especially concerning the wild interest in exosolar water-planets. The planned E-ELT that is to search for exoplanets being even further remote should be able to provide fantastic insights into this topic.

9. Outlook!

While we were waiting for a “date” with Jupiter in the morning hours of the last period of clear skies in Northern Germany (mid February 2008), we directed our telescope to Capella (alpha Aurigae). We knew about its status as a spectroscopic binary star and were interested in what would happen to its sodium-doublett. Here is what we found out (– instead of Jupiter`s blueshifted lines; we haven’t been able to measure them, simply because of the morning haze layer on the north-eastern horizon):
The two graphs (labled only in German but easily understandable for astronomers) show the Doppler shift of the Fraunhofer D1 and D2 lines in the form of separation. If time and weather permits, we are eager to plot the radial velocities during the next 104 days of Capella’s orbital revolution. This will be a good “infotainment” while waiting for the ideal Jupiter shot.



Aufnahmen am 15.2.2008
mit Maksutov-Teleskop 200/3200
Lichtleitfaser 125/100 µm
Gitterspektrometer 600 L/mm 2.Ordnung
Kamera MEADE DSI II pro
Belichtung 8 min, Spalt 17 µm

CAPELLA_NaD_AUFSPALTUNG.CDR

Kind regards!

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